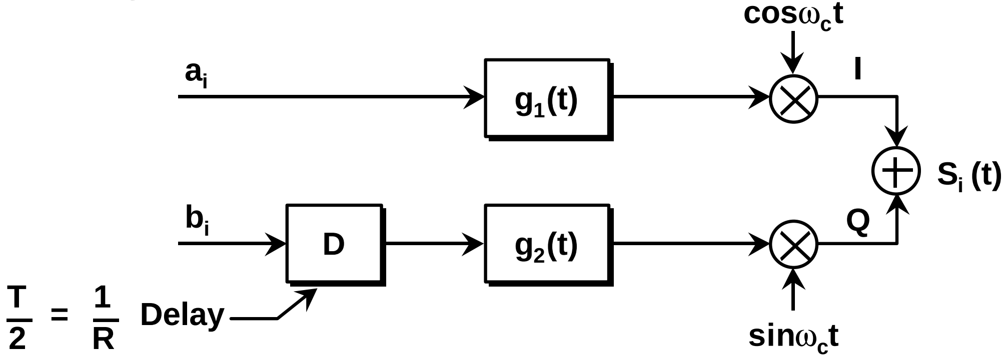
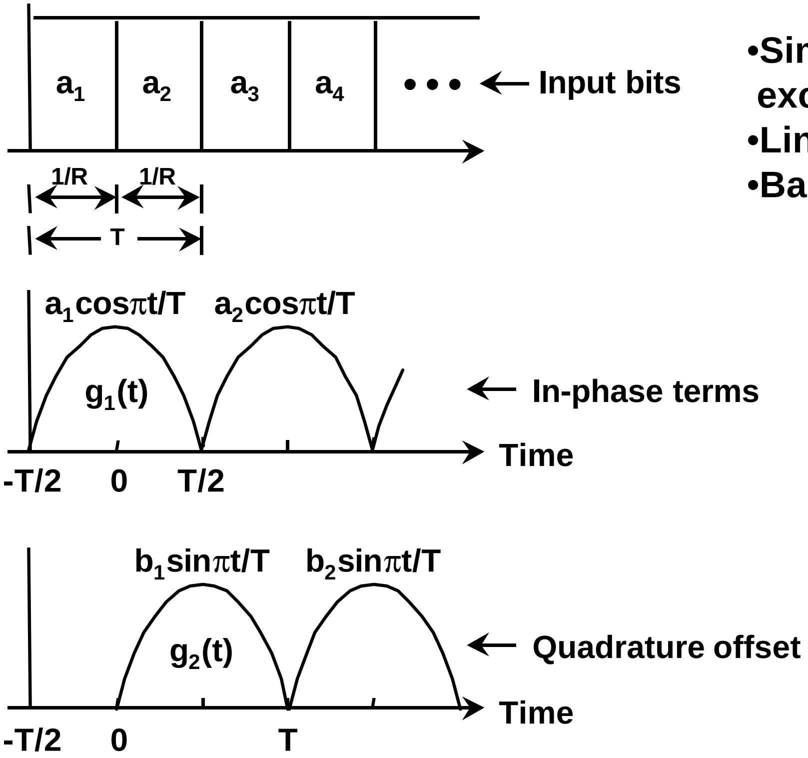
Minimum Shift Keying [MSK] Modulation

---a special form of FSK

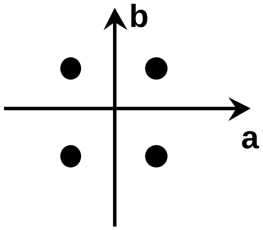
- **MSK is generated as follows**





- Similar to generation of OQPSK--- except for different I/Q baseband filters
- Linear modulation ---> can equalize
- Bandwidth determined by $g_1(t)$ and $g_2(t)$

Detect as SQAM signal



Not a strictly bandlimited signal---since time limited pulses are used for shaping

MSK is an FSK Signal

- **The MSK signal can be written as**

$$s_i(t) = \cos\left(w_c t \mp \frac{pt}{T}\right) \qquad a_i = 1,\ b_i = \pm 1$$

$$= \cos\left(w_c t \mp \frac{pt}{T} + p\right) \qquad a_i = -1,\ b_i = \pm 1$$

$$s_i(t) = \cos\left(w_c t - \frac{a_i b_j p t}{T} + q\right) \qquad q = 0 \text{ if } a_i = 1$$

$$q = p \text{ if } a_i = -1$$

- **The MSK signal has a constant envelope**
- **As shown below, MSK may be interpreted as a form of FSK**

$$s_i(t) = \cos(w_c t + \Delta w t + q) \qquad q = 0 \text{ if } a_i b_i = -1$$

$$= \cos(w_c t - \Delta w t + q) \qquad q = 0 \text{ if } a_i b_i = 1$$

$$\text{where} \quad \Delta w = p / T, \quad \text{or } \Delta f = \frac{\Delta w}{2p} = \frac{1}{2T} = \frac{R}{4}$$

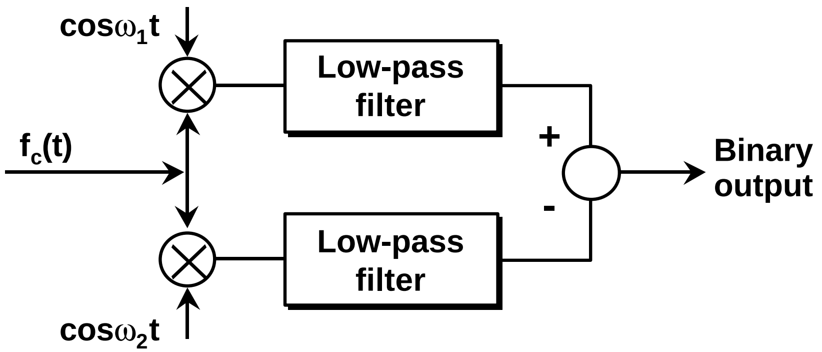
Why Is It 'Minimum' Shift Keyed?

- For the synchronous or coherent FSK detector shown below, using an integrator (over a symbol period) as the linear filter, it can be shown that the closest that the two frequencies can be spaced (for the signals to be unambiguously resolved in the absence of noise) is R/4
- If the tranmitted signal is the lower frequency, then, to a first approximation, the output of the bottom integrator is

$$\int_0^{1/R} f_c(t)\cos \omega_2 t\,dt = 1/2\int_0^{1/R} \cos(\omega_1 - \omega_2)t\,dt$$

- For this integral to be zero, we have the condition

$$\sin\left(\frac{2\Delta\omega}{R}\right) = 0 \qquad \Rightarrow \Delta f = R/4$$

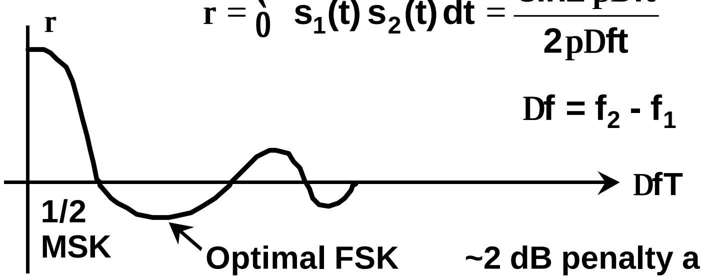


Consider $S_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_1 t$

$S_2(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_2 t \quad f_1 + f_2 = n\cdot\frac{1}{T}$

$\rho = \int_0^T s_1(t)\,s_2(t)\,dt = \frac{\sin 2\pi\Delta f t}{2\pi\Delta f t}$

$\Delta f = f_2 - f_1$



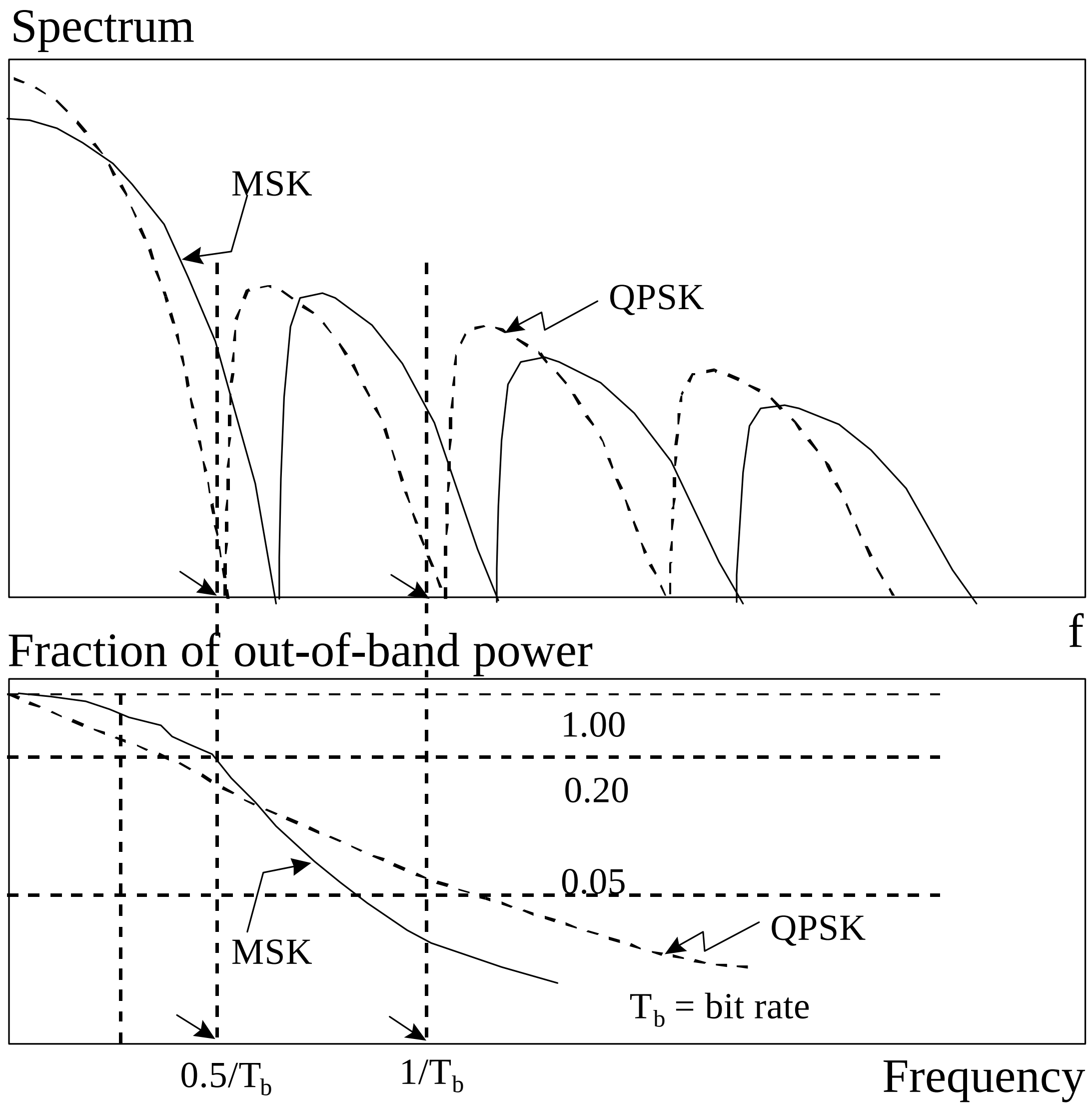
1/2 MSK

Optimal FSK
$\Delta fT = .715$

~2 dB penalty antipodal
(difficult to synchronize)

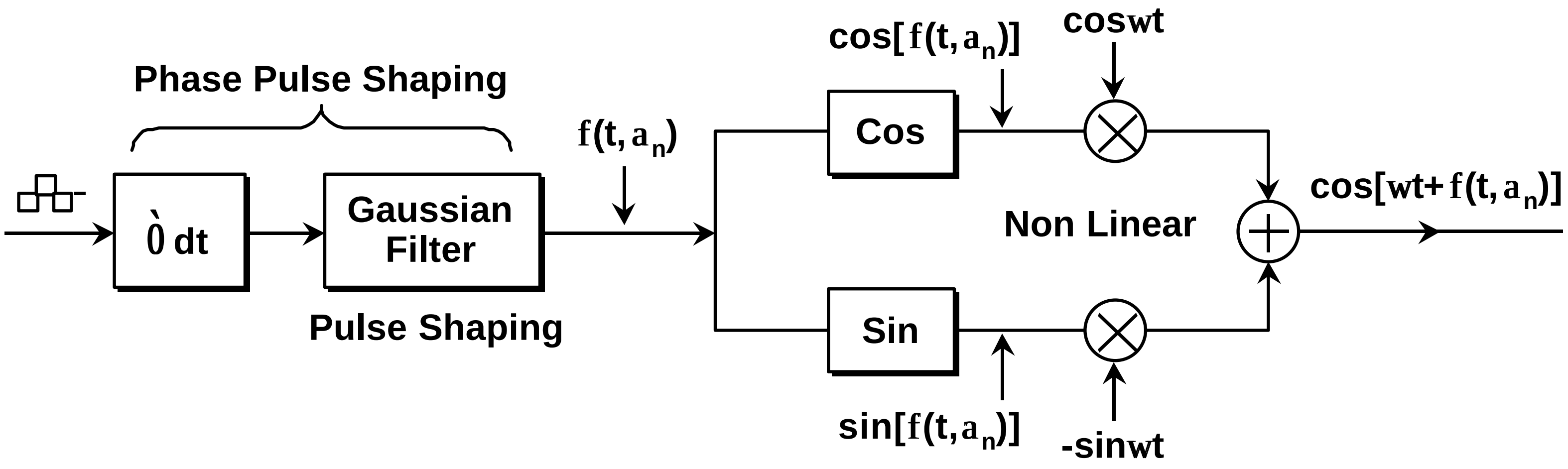
ρ = inner product between $s_1(t)$ and $s_2(t)$; measures the degree of "orthogonality" between the 2 signals

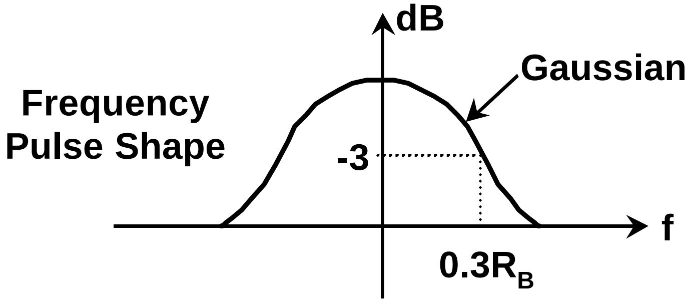
POWER SPECTRA FOR MSK AND QPSK



MSK has lower side lobes than rectangularly filtered QPSK, as shown, and the out-of-band power is significantly lower. A measure of the compactness of a modulation spectrum is the bandwidth which contains 99% of the total power of the signal, 1.2/T_b for MSK.

GMSK Modulator: Gaussian Filter Precedes $g_1(t)$ and $g_2(t)$ and reduces the bandwidth [relative to MSK]



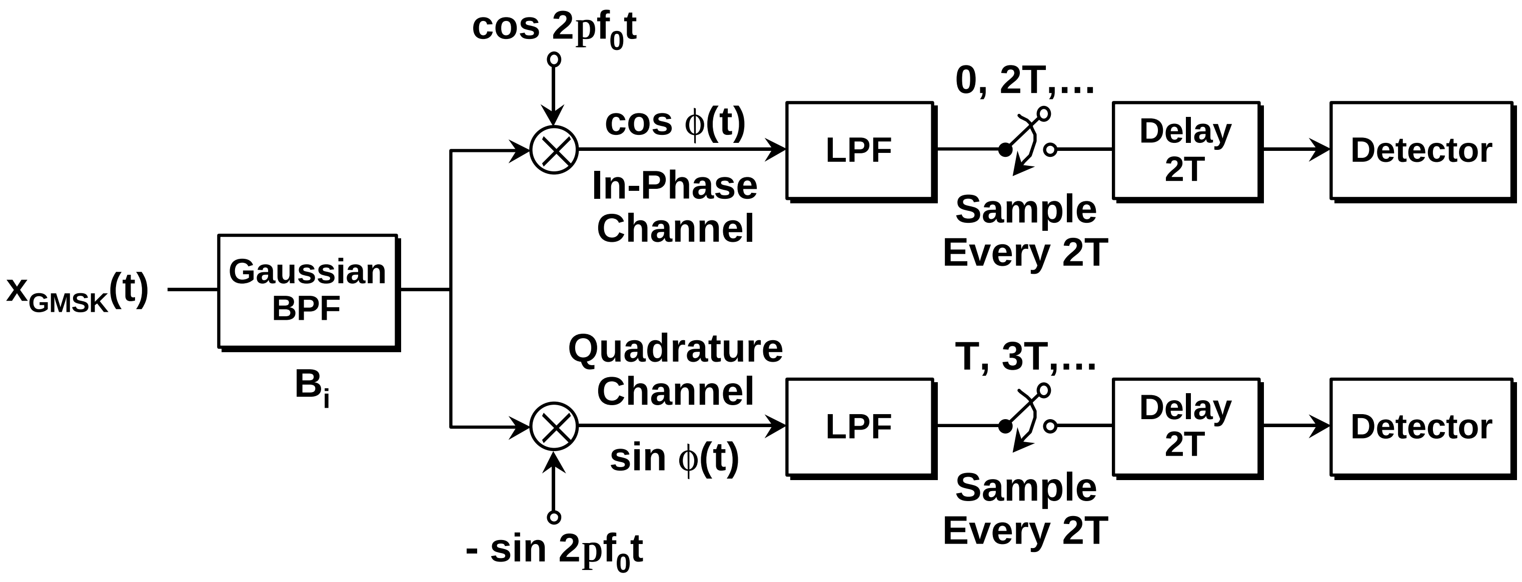


- Gaussian filter produces an even faster drop off in high frequency content than MSK
- No longer a linear modulation [like MSK]: since the baseband pulse is not a cos/sine time function
- Good approximation for MSK as a linear signal, so can equalize

GMSK Coherent Receiver

Laurent, IEEE Trans. On Communications, February 1986

Can Detect GMSK as a Pair of Antipodal Quadrature Signals ---- process as linear modulation: good approximation



- **LPF eliminates $2f_0$ terms**
- **Filtering is accomplished in BPF**